# FORMABILITY PREDICTION OF DUCTILE MATERIALS USING A NON-ASSOCIATIVE PLASTICITY MODEL AND BIFURCATION-BASED CRITERIA

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## 1 Introduction

Plastic instabilities such as diffuse or localized necking may occur during sheet metal forming processes, thus limiting sheet metal formability, which is detrimental to industry. The formability of sheet metals is usually characterized by the concept of forming limit diagram (FLD), which was first proposed by Keeler and Backofen [1] and Goodwin [2]. The FLD reports combinations of in-plane major and minor strains, thus delimiting the plane into two zones: a safe zone and a critical one located above the FLD. It remains however that the experimental determination of FLDs is difficult, time consuming and involving non-negligible costs. To overcome these drawbacks, significant efforts have been devoted in the literature to develop theoretical criteria able to predict the formability limits of sheet metals, which are associated with the occurrence of diffuse or localized necking. For reliable predictions of sheet metal formability, one of the requirements is to develop an integrated approach coupling advanced constitutive models, capable of accurately reproducing the key physical phenomena that occur during forming processes, with theoretically well-founded necking criteria.

In this work, a non-associative elastic–plastic model, with Hill'48 anisotropic plastic yield surface, is coupled with the continuum damage mechanics theory based on the Lemaitre isotropic damage model. The resulting constitutive model is then combined with four bifurcation-based criteria, namely: General Bifurcation (GB) [3] and Limit-Point Bifurcation (LPB) [4], for the prediction of diffuse necking, and Loss of Ellipticity (LE) [5] and Loss of Strong Ellipticity (LSE) [6], for the prediction of localized necking. The complete approach is implemented into the finite element code ABAQUS/Standard, within the framework of large strains and plane-stress conditions. A comparative study of the above bifurcation criteria is carried out on a mild steel, in order to classify them with respect to their order of prediction of critical necking strains.

## 2 Results and conclusions

The present approach is applied for the prediction of FLDs associated with the occurrence of diffuse and localized necking for DC06 mild steel. The material parameters corresponding to a non-associative anisotropic elastic–plastic–damage model with mixed hardening (isotropic and kinematic) are taken from [7].

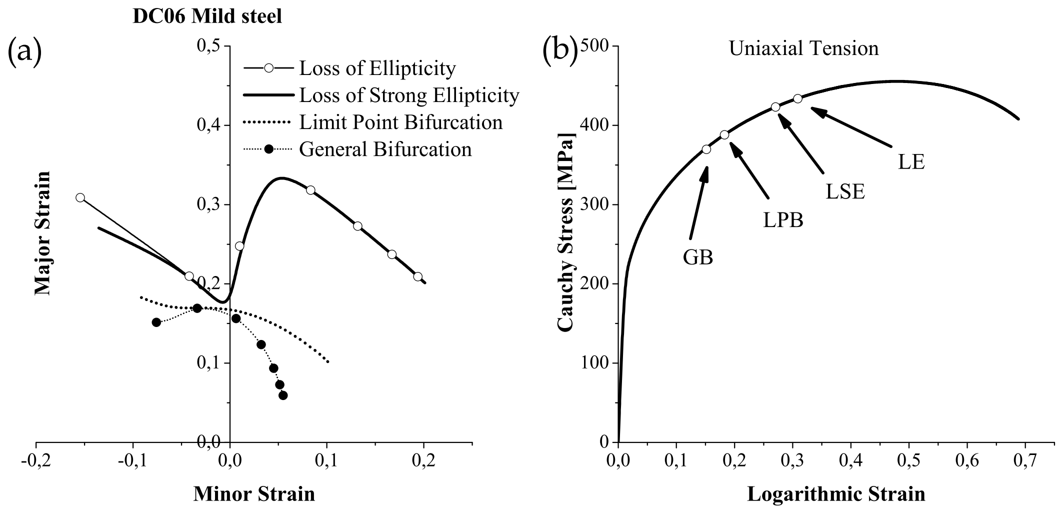


Figure 1: FLDs predicted with the bifurcation criteria (a), and uniaxial tensile Cauchy stress–strain curve (b) for DC06 mild steel

Figure 1(a) shows the predicted FLDs, which are associated with the occurrence of diffuse and localized necking using the bifurcation criteria, for the studied material. It can be observed that the predictions of necking strains, for the occurrence of diffuse necking using the GB and LPB criteria, are different for most loading paths, with the GB criterion as being the most conservative. For localized necking, the limit strains predicted by the LSE criterion are lower than those given by the LE criterion for strain paths close to uniaxial tension, while their predictions coincide in the right-hand side of the FLD. These simulation results confirm the theoretical classification established between the four bifurcation criteria, which states that the GB criterion represents a lower bound, while the LE criterion sets an upper bound. In addition to the FLD predictions shown in Figure 1(a), the points marking the onset of necking, as predicted by the different bifurcation criteria, are reported in Figure 1(b) on the Cauchy stress–strain curve, for the particular case of uniaxial tensile strain path. This figure reveals that strain localization occurs in the positive hardening regime, when predicted by the LSE and LE criteria in conjunction with a non-associative plasticity model, while it requires softening with associative plasticity (see, e.g., [5,7]).

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